Exercise 1 - Up-to-date or Behind

Alex is taking a bioinformatics class and in each week he can be either up-to-date or he may have fallen behind. If he is up-to-date in a given week, the probability that he will be up-to-date (or behind) in the next week is 0.75 (or 0.25, respectively). If he is behind in the given week, the probability that he will be up-to-date (or behind) in the next week is 0.5 (or 0.5, respectively). If we assume that these probabilities do not depend on whether he was up-to-date or behind in previous weeks, we can model the problem using a Markov chain.

a) Draw a Markov chain for this problem, showing the states and transition probabilities. Also show the transition matrix.

b) Assume Alex is up-to-date in the first class - what is the probability that he is up-to-date two classes later? What is the expected probability that he is behind after an infinitely long semester?

c) Give the transition probability matrix product for $\lim_{t \to \infty} P^t$.

Exercise 2 - Stationary Distribution

Consider a three-state Markov chain having the following transition probability matrix

$$ P = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix} \tag{1} $$

In the long run, what proportion of time is the process in each of the three states?

Exercise 3 - Reversibility

Consider a three-state Markov chain having the following transition probability matrix

$$ P = \begin{pmatrix} 0 & 1 & 0 \\ 1/3 & 0 & 2/3 \\ 0 & 1 & 0 \end{pmatrix} \tag{2} $$

a) Draw the Markov chain for this problem.

b) Given the stationary distribution $\pi^* = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$, is this Markov chain reversible? And what does this property tell you?